A Blended Multisite Distance Workshop in Mathematics Using Inquiry, Technology and Collaboration: Discrete Mathematics and Statistics

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Abstract
This paper reports the results of a blended distance learning workshop in discrete mathematics and statistics from 2009. The target audience for the workshop was 8th – 12th grade mathematics teachers. The workshop integrated technology, inquiry, and collaboration. The pretest scores indicated that most of the participants had a serious deficit in knowledge of the content. Their scores increased significantly on the posttest but not to a sufficient level; this points to a need to continue offering this type of workshop but subdividing the content from this workshop into two separate workshops. Additional assessments indicated that the participants increased their skills both significantly and sufficiently towards the pedagogical goals of the workshop.

1. Introduction

A blended distance learning workshop in discrete mathematics and statistics was administered from a university in northeast Ohio during the spring, summer and fall semesters of 2009. This was the second of two workshops with a target audience of 8th – 12th grade, inservice, mathematics teachers. The agency which sponsored the grant for the workshops stipulated the content of both workshops. The objective of the second workshop was to have the participants learn how to integrate inquiry, technology and collaboration when teaching the content of the workshop: discrete mathematics and statistics. The previous workshop was the first time that the coordinators offered a workshop with a blended distance delivery format involving multiple sites. In addition to adapting to the new mode of delivery, the coordinators also needed to adapt to a new course management system at the university and the TI-Nspire calculator/software. The structure and framework of both workshops were the same; an overview will be given in this article but the more details refer to *A Blended Multisite Distance Workshop in Mathematics Using Inquiry, Technology and Collaboration: An Initial Report* [4]. The content focus of the first workshop was on topics prerequisite to calculus. The posttest scores indicated that the participants improved to a sufficient skill level by the end of the workshop. This was not the case with the second workshop. Even though the increase in test scores was similar between the two workshops, the scores on the pretest for the second workshop were so low that the increase did not bring the average above a failing level. The results did however show that the participants increased their skills sufficiently towards the pedagogical goals of the workshop.

2. Purpose

Spurred by the accessibility offered by new technology, the fields of discrete mathematics and statistics have been growing in importance and hence there has been a growing demand for courses in these subjects at the high school and college level (for discrete see [3] and [9]; for statistics see [1] and [2]. Technology makes it effortless to manage the cumbersome computations that permeate applications in discrete mathematics and statistics; hence applications that have been traditionally reserved for advanced postsecondary courses are now accessible in lower level college and secondary courses. Cobb [2] notes that the ratio of statistics to calculus courses in two-year colleges has increased from 1 to 10 in 1966 to about 1 to 5 in 1990, yet teachers often lack the content and/or pedagogical knowledge for teaching these topics (for discrete see [10]; for statistics see [5], [8], [2]). The primary goal of this workshop was to provide a means for inservice teachers to update both content and pedagogical knowledge for discrete mathematics and statistics. This included integration of the latest hand-held technology. One of the benefits of the blended distance format was to allow teachers at remote sites to enroll in the workshop. The topics from the workshop included statistical charts, central tendency and spread, random variables, distributions (normal, binomial, chi-squared), hypothesis testing, data analysis, regression, matrices, systems of equations, set operations, counting, permutations, and combinations (with and without repetition).

3. Theoretical Framework

Five pedagogical strategies were employed continuously throughout the workshops: technology integration, inquiry-based (IB) instruction/activities, peer collaboration, reflection and formative assessment. All of these strategies have been applied effectively to mathematics instruction in general and in a distance learning format (for more details see [4]). As mentioned earlier, one of the reasons for the increasing importance of discrete mathematics and statistics is their connection to technology; hence technology-supported activities naturally lend themselves to instruction in these areas. Additionally, the American Statistical Association recommends using active learning, technology, and formative assessment in teaching [6]. According to Fernandez and Liu [5], incorporating technology, activities, and cooperative learning motivates students and leads to effective learning of statistics. Small group activities led to improvements in students attitudes when learning statistics [7]. These five strategies complement and support each other in a number of ways. For example, technology facilitates inquiry-based investigations of problems involving data, collaboration makes these problems easier to tackle, and informative assessment helps students reflect on their learning. Combining these teaching strategies helped to form a more comprehensive pedagogical design.

4. Methodology

4.1 Course Logistics

The workshop classes took place in the spring and summer of 2009, with a follow up meeting in October. Sixty-nine mathematics teachers of grades 7 through 12 participated in the entire workshop. The original workshop enrollment was 76; however, 3 teachers opted to complete only the first 3 credits of the workshop and 4 teachers had to withdraw for personal reasons. The participants came from 37 school districts, 27 high schools, and 11 middle schools. Participants that completed the first 6 days of the workshop earned 3 graduate credits and those that completed the entire workshop earned 6 graduate credits. Tuition was covered by the University through scholarships; leaving participants owing approximately $420 to cover fees. The workshop spanned 12 days: five 6.5 hour meetings on Saturdays during the spring semester, six 8.5 hour meetings during the summer and a 4-hour follow-up session on a Saturday in October.

The workshop involved five sites located in five northeastern Ohio counties. The course instruction originated from the university where the workshop organizers worked. There was a facilitator employed at each of the four satellite sites. Three of the facilitators were instructors and one of them was a graduate students; each of whom had previous experience in helping with this type of training. All of the sites were equipped with cameras and microphones for interacting in real-time by means of videoconferencing. SMART Boards and "See and Share" software were used to transfer control for interactive presentations between the sites.

The workshop utilized the course management system Springboard, which is authored by Desire2Learn Inc. Participants continuously accessed this website in and out of class. Each of them had a laptop with wireless access available during the classes. All of the course materials such as the syllabi, activities, readings, activity reflection forms, and rubrics were posted online under the Content section of Springboard. At the end of every class, participants were emailed a list of everything that was due, using the Classlist feature on Springboard. Typically, a daily reflection was due by midnight, any debriefed activities and their reflections were due the next day by noon, and they were usually given two new activities to prepare for debriefing during the next meeting. Participants regularly took advantage of the asynchronous, anonymous, threaded discussion which allowed them to post questions and comments as they worked on assignments. A drop box was created on Springboard for each activity solution, activity reflection, daily reflection, reading synopsis, and for the final project as well. The facilitators would download the posted solutions and reflections for grading and then upload them to each team's personal Locker on Springboard, with comments and grades, usually within a day of the next meeting. This type of formative assessment helped the participants to gauge their progress in this fast-paced workshop. Besides tracking their own grades, participants could also view class statistics – such as means, medians, minimums, maximums, and ranges – for every graded item.

Throughout the first workshop, the TI-Nspire calculator was the key means for introducing new approaches for solving problems via IB activities. This particular calculator had been difficult for the participants to adapt to during that workshop, even though many of the participants and facilitators regularly used other calculators from Texas Instruments. One of the issues centered on becoming adept at manipulating the sensitive calculator keys and another was learning how to toggle back-and-forth between calculator screens and accessing the tools available for each platform. During that workshop the facilitators found that the TI-Nspire software was much easier to learn because using the computer mouse made it easier to switch between platforms and to locate the tools available for each environment. Thus, using the software, one could focus on learning their way around the TI-Nspire operating system. Once knowledgeable with the operating system, one could focus on adapting to the physical limitations of the calculator. Because of this it was decided that participants would start with the software and gradually transfer that knowledge to using the calculator during the new workshop. Happily, as evidenced by participants' comments, this change simplified the process of introducing the new technology.

4.2 Course Structure

On the first day of the workshop, each of the five sites was partitioned into 2 or more groups of size 3 to 5 for a total of 19 groups (with an average of 4 students per group). The most important factor in forming the groups was to put participants together who taught similar courses. At times this was not feasible, especially because two of the sites had only 7 participants each. During the first workshop there was an attempt to improve this by forming groups across the sites, however, there was strong opposition to this from the participants; they wanted to be teamed with people in the same room as them during the classes. The participants worked with their groups on a total of 21 IB activities and a final project.

Groups took turns debriefing the activities. Facilitators and participants at any of the sites could ask questions or request pauses in the discussion during these debriefings. This policy helped everyone keep up with the work and/or add their individual observations, although it also led to some overly lengthy debriefings. After each debriefing, the groups were given time to upload their reviewed activity solutions and reflection. The reflection included mapping the activity to Ohio Department of Education indicators for mathematics as well as comments on how they could use what they had learned in their classroom.

Besides experiencing technology integration, IB activities, and peer collaboration as learners, the participants were also required to employ these strategies in the lessons that they created for their final project. Having the participants engage in these strategies while learning about them increased the likelihood that they would employ them in their own classrooms (NCTM, 1991). In the previous workshop, to prepare for the final project, participants read and discussed a chapter on IB learning in mathematics. Then, after completing 21 IB activities they were expected to construct their own IB lessons, consisting of two or more IB activities, for the final project with the help of an outline and tip sheet. Before turning in the final version, the projects were exchanged amongst the groups for an anonymous peer critique. They found this feedback to be very helpful. However, in the follow up survey many expressed a need for more instruction on creating the IB lessons. So, in the new workshop they were also given two sample IB-lessons to critique and improve before being asked to create their own. They thought that this was helpful as well, but still wanted more time creating lessons and/or adapting workshop activities for their own needs. At the end of the workshop, the lessons from their final projects were available to all of the participants via Springboard. The grading rubric that they were given in advance can be found at Appendix A.

4.3 Course Content

The content of the workshop was expansive because the goals set by the sponsoring agency were ambitious. This made it necessary to have most of the IB activities assigned as homework and then debriefed during class time. Two of the IB activities are located at Appendices B and C. One of these is over regression and the other is over statistical charts. The content of the workshop is outlined in the rest of this section.

Some of the participants, including those who had taken the first workshop, had already been exposed to the new TI-Nspire calculators. So, whenever possible, at least one of these individuals was placed on each team to act as mentor for those who had no experience with the new calculator. As mentioned earlier, the TI-Nspire software was used to acclimate the participants to the operating system. At the start of the workshop, participants were given a brief introduction to the different applications and how to access them, but then the focus quickly turned to the "Calculator" and "Graphs and Geometry" applications along with the tools available within them. This was done because most of the participants were familiar with these applications as they already existed on the TI-83 and TI-84 calculators to which most of them were previously accustomed. This predisposition eased the transition to new applications and tools that were introduced on an as needed basis.

Data analysis began with a brief introduction to the concept of regression and how to judge the "goodness of fit" both graphically and numerically. It was emphasized that regression is now used prevalently in courses prerequisite to calculus through numerous examples containing real data that could be modeled with every family of continuous functions. After being led through a couple of examples on linear and nonlinear regression using the application "Lists and Spreadsheet" together with the application "Graphs and Geometry" they were ready to discover many relevant concepts via IB activities. These activities illuminated concepts connected to the effects of outliers, the med-med line and when it is appropriate to use it, and the possibility of not having the best fit despite having an excellent regression coefficient. Along the way they were introduced to the "Data and Statistics" application. Next they learned how to use the residuals plot when judging a particular model via the size and randomness of the residuals. They were also briefly introduced the concept of linearization.

With systems of linear equations they learned the connection between the solution set and the row reduction process in two and three dimensions; this included the geometrical interpretation. Also, there was an emphasis on the importance of using the reduced row echelon form of the augmented matrix to decide whether the system is consistent. Although most of the teachers had taken linear algebra, many were not fluent with recognizing when a system is inconsistent or when it is consistent with infinite solutions. They also were not fluent with expressing the solutions in parametric form and then interpreting the solution geometrically. The idea of using the pivot columns to distinguish between basic and free variables was also new to many of them. This section was concluded by using linear systems to determine polynomial regressions of any degree.

About four days were devoted to discrete content that built counting techniques. More class time was spent with this content because many of the activities were introduced in the class (as opposed to the initial exposure coming from a homework assignment). This began with set operations (union, intersection, difference and complement), which naturally laid a foundation for the multiplication and addition rules of counting. A number of multiplication and addition problems of increasing difficulty were practiced. This practice led to the formulas for permutations (for r elements out of n elements) and combinations (with and without repetition).

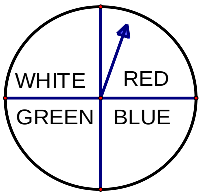
Approximately three days were devoted to statistical content. This began with constructing and interpreting frequency, bar, and pie charts, dot plots, histograms, and frequency polygons. TI-Nspire software was used for constructing these charts and advice was given for selecting which types of charts and categories would be appropriate for data of various types and sizes. The lesson on measures of central tendency and spread included calculating and understanding the significance of means, medians, modes, standard deviations, interquartile ranges, and outliers followed by interpreting and constructing box-and-whisker plots. Lessons on probability introduced the concepts of population, sample space, events (independent, dependent, and complementary), the principle of inclusion and exclusion, the law of large numbers, expected value and the difference between theoretical and experimental probability. The inverse relationship between probability and statistics was made explicit; pointing out that probability uses the population to make inferences about a sample whereas statistics use samples to make inferences about a population. This was used to segue into defining random variables (discrete and continuous) and their probability density functions. The use of the sliders in the TI-Nspire software was beneficial when investigating changes in the distribution graphs as the means and standard deviations were varied. This was followed up by using the software to estimate the areas under these graphs over confidence intervals in order to calculate probabilities for hypothesis testing. The three distributions that were examined and used for the hypothesis testing were the normal (including standardized), binomial, and chi-squared. Naturally the lessons over hypothesis testing included p-values, significance levels and Type I and II errors.

4.4 Assessment Tools

The workshop utilized an anonymous pretest and posttest, self assessment surveys, a workshop evaluation survey (performed by an external evaluator), a follow up survey (given to the participants), graded assignments, and informal observations. The assessments were devised to measure improvements in the participants' content and pedagogical knowledge, effects of the workshop in the participants' classrooms, and the effectiveness of the workshop design.

The 25 test questions were constructed by the facilitators and tested by other instructors; 14 were in a multiple choice format and the other 11 were in a short answer format. They were composed of 10 on statistics, 6 on data analysis/regression, 4 on matrices/systems of equations, and 5 on probability and counting. A sample question from each category along with average scores and t values are in Figures 1–4.

A spinner with four equal-sized sections is shown below. You decide to test whether the spinner is fair (all four colors are equally likely to occur in a given spin) using a chi-square test. The spinner is spun 100 times. White occurs 35 times, red occurs 12 times, blue occurs 19 times, and green occurs 34 times. You calculate the Chi square test statistic, X^2, and then use this value to find the p-value is .001477. Thus, you conclude that the spinner is not fair with 99% confidence. What is X^2?



3.86

15.44

96.5

115.44

386

Pretest: (M = 19%, SD = 39%) Posttest (M = 39%, SD = 49%) $t(69) = 2.89$, $p<.01$ (two-tailed)

Figure 1: *Sample of Statistics Question*

What <u>regression model</u> (linear, quadratic, exponential,…) will you tell your students to use for each of the following sets of data representing:

The initial growth of bacteria on a Petri dish?
The initial change of volume of several hardwood trees of the same species?
The fever of a sick person during a week in the hospital?
The free fall of an object on the moon?
The cost of different amounts of apples on a supermarket?

Pretest: (M = 51%, SD = 20%) Posttest (M = 63%, SD = 22%) $t(69) = 4.10$, $p<.01$ (two-tailed)

Figure 2: *Sample of Data Analysis/Regression Question*

Solve for x, y, and z using row reduction of matrices.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Pretest: (M = 43.25%, SD = 49%) Posttest (M = 68%, SD = 47%) $t(69) = 3.96$, $p<.01$ (two-tailed)

Figure 3: *Sample of Matrices/Systems of Equations Question*

An electric combination lock requires a 4-digit "pass-code" to open. The possible digits are 0 through 9.
How many possible "pass-codes" can be created if repetition of digits is allowed? **10,000**
How many possible "pass-codes" can be created if repetition is not allowed? **5,040**
Assuming repetition is allowed, how many "pass-codes" have at least one seven? **3,439**

Pretest: (M = 56%, SD = 32%) Posttest (M = 65%, SD = 31%) $t(69) = 2.39$, $p=.02$ (two-tailed)

Figure 4: *Sample of Probability and Counting Question*

Grading was split up amongst the facilitators. One of the facilitators graded all of the pretests and posttests. Each set of activity solutions and each set of activity reflections were graded by one facilitator using rubrics for the sake of consistency. The 19 final projects were split amongst the facilitators and graded using a rubric with 22 categories (which summed to 100 points).

5. Results

The results of the assessments indicate that in the area of deepening content knowledge, the goals were not met. In the areas of integrating technology, IB instruction, and group work with instruction, the assessments indicate that the workshop was successful.

5.1 Content

Same subject, paired sample t tests were performed on the results from the pretest and posttest. The questions on the tests were broken into four non-overlapping categories and t tests were performed on each of them as well as the entire test. The assumptions for using t tests were checked by examining the plots of the differences between the pairs. The plots were approximately normal with the exception of the probability and counting category; this category contained a couple of moderate outliers. As a precaution, the Wilcoxon Signed Rank test was performed and the results supported the t test results for this category. The results are summarized below:

- The mean score on the posttest (M = 54.63%, SD = 16.53%) was significantly higher than the mean score on the pretest (M = 37.88%, SD = 15.11%), $t(68) = 11.17$, $p < .001$ (one-tailed). The mean score on the test increased by 16.75%.
- The mean score of the statistics questions on the posttest (M = 44.80%, SD = 18.40%) was 9.62% higher than the mean score on the pretest (M = 35.18%, SD = 17.97%), $t(68) = 4.62$, $p < .001$ (one-tailed).
- The mean score of the data analysis/regression questions on the posttest (M = 68.60%, SD = 20.86%) was 30.13% higher than the mean score on the pretest (M = 38.47%, SD = 18.48%), $t(68) = 12.90$, $p < .001$ (one-tailed).
- The mean score of the matrices/systems of equations questions on the posttest (M = 56.30%, SD = 28.95%) was 29.00% higher than the mean score on the pretest (M = 27.30%, SD = 26.92%), $t(68) = 8.83$, $p < .001$ (one-tailed).
- The mean score of the probability and counting questions on the posttest (M = 58.15%, SD = 23.63%) was 6.56% higher than the mean score on the pretest (M = 51.59%, SD = 22.87%), $t(68) = 2.53$, $p = .007$ (one-tailed).

The increase on the test average was similar to the increase of 18.5% from the first workshop [4]. Even though the increase on the new test was significant at the .01 level, it was not enough because the average score on the posttests was not at or above a 70%. Only the category of data analysis/regression approached an average of 70%. This suggests that having one workshop over these four content areas was not sufficient. Perhaps two workshops, one over statistics and one over the other three categories, would be enough for the participants to develop a sufficient understanding of the content.

5.2 Pedagogy

The participants were given self assessment surveys related to the group work, IB learning, and technology used in the workshop. The participants reported statistically significant (at the .05 level) gains in all of these areas. The gains connected to the technology were the most substantial. The results are shown in Table 1.

Table 1: *Participant Self Evaluation Survey Related to Workshop Strategies*

Likert Scale: 1 Lowest through 5 Highest	Presurvey	Postsurvey	Difference
1. Experience <u>as a student</u> participating in cooperative groups	3.72 (0.82)	4.29 (0.73)	.57
2. Proficiency using inquiry-based lessons in your teaching	3.07 (0.81)	3.85 (0.68)	.78
3. Proficiency developing inquiry-based lessons	3.04 (0.85)	3.79 (0.69)	.75
4. Familiarity with the TI-Nspire software	1.86 (1.00)	3.33 (0.85)	1.47
5. Familiarity with Springboard	2.06 (1.34)	4.00 (0.75)	1.94

Note. Standard deviations are in parentheses.

During the previous workshop there was a strong consensus that the goals related to peer collaboration were met. Evidence for this came from a group work survey, general workshop survey, facilitator observations, and an appraisal given by an external evaluator. Because of the strong positive results, it was deemed unnecessary to give the special group work survey in the new workshop, as it seemed enough to rely on the other three measures. Once again, all of the indicators were positive. They reported that their experience in group work increased (see question 1 in Table 1). The group work scores from the workshop ranged from 50 – 100% (excluding days when a participant was absent) with an average of 96.51% (including days when a participant was absent). The external evaluator summed up the results well in his report "Teachers were thankful they were in groups during the class because they were better able to do homework. They also indicated group work was essential to the learning process."

In preparation for writing their own IB lesson plans, the participants read and wrote a synopsis of Math Inquiry: Developing Curious Students by authors Jenny Tsankova and Galina Dobrynina, a chapter from the book *Integrating Inquiry Across the Curriculum* (edited by Richard H. Audet and Linda K. Jordan) [11]. Four of the synopses received a score of 2 out of 4, three received a score of 3 out of 4, and the remaining 64 received a score of 4 out of 4 for an average of 96.13%. After discussing this reading, the participants were given two sample IB lessons to critique. They were on the same topic and one of them was purposefully lacking in inquiry. There was a discussion about the strengths and weaknesses of each of them as well as how they could be improved using an outline and tip sheet for IB lessons. In the follow up survey, participants wrote that this process was helpful for learning how to write IB lesson plans. This was also evident in their increased self-ratings on questions 2 and 3 in Table 1. Each of the self-ratings on understanding and using IB methods increased by about 0.75 points. The rubric for grading the final project included nine categories demonstrating the IB techniques from the reading. These categories were worth 34% of the project grade. The inquiry scores from the project ranged from 80 – 100% with an average of 93.28%.

According to the external evaluator "The consensus was clear that the participating teachers learned how to write inquiry-based activities for their students". Yet, he found that this was not exactly the case with the middle school teachers because some of them believed that they could not use the lessons that they wrote in the workshop since their students did not have access to TI-Nspire calculators. Apparently, they were not comfortable with altering the lessons to take advantage of the calculators to which they did have access. Some of the participants expressed a strong interest in spending more time creating lessons for their classes in the follow up survey and with the external evaluator. Perhaps more time spent on creating IB lessons that incorporate the available technology at their particular school, would increase the usage of IB lessons in the classrooms of future workshop participants. However, the participants improved significantly towards the goal of writing and using IB lessons overall and according to the external evaluator "Relative to the value of inquiry-based learning, everyone agreed that it is preferable to direct teaching".

As mentioned earlier, the TI-Nspire software was the technology used for establishing new approaches for solving problems, with a gradual transfer to the TI-Nspire calculators. The self ratings, in familiarity with this software, increased from 1.86 to 3.33 as seen in question 4 in Table 1. According to the external evaluator "As for learning how to use the TI-nspire CAS, the consensus was that the on-line lessons were sufficient, but some would have liked more face-to-face instruction. I sensed that using the nspire went well, with some indicating a preference to use the computer version. Unlike last year, no one complained that the technology was too difficult or that it was something they would not use in their classrooms". This was corroborated by the follow up survey where participants stated that they were comfortable using the software, but that they wanted more time spent on the calculator. The curriculum of the workshop was crowded, yet if the content could be split between two workshops as previously mentioned, it would be possible to spend more time on calculator instruction. The rubric for grading the final project had a category on integration of the technology which was introduced in the workshop. This category was worth 10% of the project grade. The technology scores from the project ranged from 65 – 100% with an average of 93.45%. These results indicate that the participants improved significantly towards the goal of using the technology introduced in the workshop.

Surveys were also given to ascertain what type of technology was supported by the participants' mathematics textbooks, what type of technology they used, and how often technology was employed in workshop content. The results in Table 2 indicate that the participants rated their technology integration as higher than that of their textbooks: particularly their use of scientific calculators.

Table 2: *Participant Self Evaluation Survey Related to Type of Technology Utilized*

The following Likert scale was used on these questions: 1=Not at all 2=very little 3=some 4=often 5=Continuously	Book	Self
1. Does the book you use in your most advanced class integrate the use of technology in the teaching/learning of mathematics? Do you integrate the use of technology in the teaching/learning of mathematics in your classes?	2.84 (1.01)	3.55 (0.91)
If you answered 2, 3, 4 or 5 to the previous question, please rank how often that book integrates each type of technology. If you answered 2, 3, 4 or 5 to the previous question, please rank how often you use each type of technology.		
Scientific Calculators	3.25 (1.35)	3.83 (1.25)
Graphing Calculators	3.23 (1.37)	3.12 (1.38)
Math Software	1.88 (1.01)	1.94 (1.29)

Note. Standard deviations are in parentheses.

Table 3 compares how often technology is used on the workshop content between the participants' mathematics textbooks, the participants themselves, and their anticipated use after the workshop. There is a striking similarity between how the textbooks and teachers apply technology. The mean score for the textbooks on questions 2 – 16 (M = 2.05, SD = 0.35) was nearly identical to the corresponding mean score for the teachers on the presurvey (M = 2.07, SD = 0.40). The strong correlation (R = 0.983) between how the textbooks and the teachers use technology, $p < .001$ (two-tailed), suggests that the teachers are strongly influenced by their textbooks in technology usage. However, the teachers rated themselves noticeably higher (by 0.71) on question 1 in Table 3, related to how much they use technology. The reason for this may be related to their response to question 2 in Table 2. Here they rated themselves 0.58 higher in usage of scientific calculators than their textbook. It is likely that they would be using graphing calculators in the applications addressed in questions 2 – 16 of Table 3. This implies, more specifically, that the teachers are strongly influenced by their textbooks in technology usage related to their graphing calculators.

The participants increased their self ratings on each of questions 2 – 16 in Table 3 by about 1 point on average. Each of these increases was significant at the .05 level. The presurvey mean scores (M = 2.05, SD = 0.35) were centered near the "very little" usage category and the postsurvey mean scores (M = 3.00, SD = 0.48) were centered on the "some" usage category. This is a good indication that the teachers will integrate technology more often when teaching the content of the workshop.

In the follow up survey, many of the participants reported that they had already tried out inquiry-based lessons, technology, group work, and reflections in their classrooms during their first month back at school in September. They reported trying activities involving permutations, combinations, probability and counting methods. Some said that they had plans to implement more but had not had the opportunity yet.

Table 3:*Participant Self Evaluation Survey Related to Type of Technology Applications*

The following Likert scale was used on these questions: 1=Not at all 2=very little 3=some 4=often 5=Continuously	Book	Pre	Post
1. Does the book you use in your most advanced class integrate the use of technology in the teaching/learning of mathematics? Presurvey: Do you integrate the use of technology in the teaching/learning of mathematics in your classes? Postsurvey: Will you integrate the use of technology in the teaching/learning of mathematics in your classes?	2.84 (1.01)	3.55 (0.91)	4.10 (0.88)
If you answered 2, 3, 4 or 5 to the previous question, please rank how often that book integrates each type of technology. Pre survey: Using the scale below, rate how you address the following topics using technology. Post survey: Use the scale given below to rate your expected use of technology in each of the following topics.			
2. The use of nontraditional tools such as lists, sequences, recursion to solve different problems?	2.30 (1.08)	2.25 (1.15)	3.33 (1.00)
3. Modeling real data using linear regression (LSL)	2.16 (1.17)	2.12 (1.23)	3.22 (1.16)
4. Discussing the effect of outliers and what to do when they are present?	2.38 (0.97)	2.51 (1.06)	3.42 (0.88)
5. Re-expressing data?	2.01 (0.90)	1.93 (0.98)	2.79 (1.15)
6. Modeling real data using nonlinear regression for each family of continuous functions studied?	2.19 (1.11)	1.97 (1.09)	3.13 (1.11)
7. Analyzing and/or comparing regression models via residuals	1.64 (0.80)	1.63 (0.91)	2.75 (1.17)
8. Modeling with matrices (Networks, Markov, Transformations…)?	1.87 (0.91)	1.84 (1.02)	2.88 (1.23)
9. Solving m x n systems of linear equations	2.35 (1.16)	2.47 (1.22)	3.18 (1.15)
10. Using recursion in the calculator home screen?	1.46 (0.70)	1.55 (0.88)	3.01 (1.20)
11. Modeling recursive problems?	1.65 (0.89)	1.60 (0.98)	2.91 (1.21)
12. Solving linear systems using row reduction?	1.81 (1.07)	1.79 (1.09)	2.84 (1.23)
13. Solving linear systems using Cramer's rule?	1.87 (1.17)	1.82 (1.15)	2.40 (1.20)
14. Probability	2.71 (0.97)	2.82 (1.06)	3.49 (0.91)
15. Basic statistical graphs (bar graphs, pie charts, box plots,…)	2.46 (1.07)	2.63 (1.12)	3.68 (1.00)
16. Statistical Calculations (descriptive, confidence intervals, tests, distributions, …)	1.83 (1.00)	1.91 (1.18)	2.88 (1.34)

Note. Standard deviations are in parentheses.

They believed that the inquiry-based activities increased student engagement in the learning process and took the focus off of the teacher. They said their students seemed to learn more deeply with inquiry, debriefing and reflective tasks, but it also took longer for them to cover the material. Their students experienced frustration while becoming accustomed to inquiry-based activities. Some seemed more resistant because the inquiry required them to take a more active role in their learning. In particular, some were very resistant to explaining their work. Their students seemed to enjoy the group work but it was not easy to get all of the students to do their share. Also, many wanted to divulge their results before others had had a chance to finish their work. They found that their students enjoyed presentations that utilized the software and were getting more comfortable with the technology.

On the whole, the results of the assessments show that participants increased significantly in the areas of content and pedagogy. Even so, the increases in content knowledge were not enough to overcome the initial deficits in this area.

6. Conclusions

The experience gained from the first blended delivery workshop was invaluable to the success of the subsequent workshop. The content of that workshop had been presented previously using a face-to-face format in 2007. Because of that, the facilitators' main focus was on adapting to the new technology and format in 2008. In the time between that first blended workshop in 2008 and the new workshop in 2009, most of the technological and delivery issues were resolved. The main issue that is still left unresolved centers on the delivery. The participants and facilitators believed that distance learning delivery could be an obstacle to learning because at times it was difficult to see and hear what was happening at the other sites. As technology improves, this difficulty should lessen. Some of the participants believed that this issue could have been alleviated by holding the debriefings separately at each of the sites. This was not possible, however, because not all of the facilitators were fluent in the content of the new workshop.

The workshop was fast-paced for the participants. Many of them had not taught much if any of the content and may not have even been exposed to it as undergraduates. Because of this, some of the participants struggled even with the review materials. The results on the pre- and posttests for the new workshop bear this out. Whereas the scores from both years increased by about the same percent, the pretest scores were so much lower in the new workshop that the average on the posttest was still failing. One of the unfortunate tactics that some of the groups used to complete assignments was to split them up amongst themselves. This meant that many of them would not be exposed to the material until the class debriefing. Being able to break up the content between two workshops would likely alleviate this issue and bring the mean test scores on the content to a passing level. It would also create more time for expanding calculator skills and developing additional IB lessons as wished for by the participants. The pedagogical goals of the workshop were achieved and significant progress was made towards the content goals. In his report, the external evaluator concluded "The vast majority of the teachers agreed that the course was a success in that they learned new mathematics, improved their inquiry lesson-writing ability, and learned more about the related technology".

7. Acknowledgements

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Appendix A

Final Project Grading Rubric

Maximum Point Value	Assigned Point Value	Rubric for Grading the Final Project
3		Learning Objectives are clear and well defined
8		Content is correct.
7		Depth of content is appropriate.
7 IB		The lesson facilitated conceptual learning.
6 IB		Questions lead students through (at least parts of) the activity.
4 IB		There is at least one challenging problem for the students to tackle.
3 IB		The lesson connected to students' preexisting knowledge.
4		There are multiple ways given to represent problems.
4		There are multiple strategies tried for solving the problem.
3 IB		Students will need to use reasoning and proof, for example by posing and testing hypotheses.
2 IB		Students will reflect on results (by summarizing definitions & properties, considering a new or simpler solution method,…)
3 IB		Students will generalize and/or extend results.
4 IB		Students will be required to communicate (orally and/or in writing).
2 IB		Possible hints for students and teachers are listed.
10 TECH		Technology demonstrated in this workshop was incorporated in the lesson. Multiple representations using technology must be present (numerical, graphical, algebraic), using the TI-Nspire capabilities: spreadsheets, dynamic geometry software, calculator, etc
4		The activities are focused.
2		At least part of the content is relevant to the students.
3		Worksheet(s) are easy to understand.
10		Assessment is planned and described (could be through a worksheet, observation, etc.) Assessment should address the key ideas and properties in the activities.
5		Lesson satisfies requirements: for example, it was turned in on time and it followed the Inquiry-Based Lessons Format (located under Content on Springboard)
4		Grammar, spelling, mechanics, and punctuation are correct.
2		Any references are correct and sources are cited correctly.
Sum 100	Sum	

Inquiry-Based Categories indicated with IB

Appendix B

Sample of an Activity on Regression

Q. Do an excellent regression coefficient and great visual fit guarantee the best possible model for our data?

Example. Table 1 contains the average distance to the sun and the orbit's duration of each of the six planets discovered in the year 2050 in the NT2 solar system in the Andromeda's constellation. It has been conjectured the existence of another planet AP8 at a distance of 2164 millions of kilometers from the sun NT2. If this theory is confirmed, i) how long will AP8's orbit last? If there were a planet whose orbit lasted 139140 days, ii) how far from NT2 would it be?

Planet	***Average distance from*** NT2 ($\times 10^6$ Km.)	Orbit's duration (in days)
AP1	299.2	1034
AP2	1557	12287
AP3	2854.6	30508
AP4	5676.5	85565
AP5	8995.7	170735
AP6	11827	257421
AP7?	2164	?
	?	139140

Table 1. Planets of the NT2 solar system: orbits & distances

1. Find the regression line.

   ____

2. Judging by the regression coefficient obtained as well as by the visual graphical superposition of the linear and the data, how do you judge the fit on a scale of 1(=bad) to 5 (=excellent)?

   ____

3. Answer questions i) and ii) using the model found.

   i) ______________________; ii) ______________________

<u>However, how do we know that there is not a better model for the data?</u>

4. Using the given data find the following models and the corresponding answers to i).

   d) a quadratic model:__________________________

   e) a cubic model: ____________________________

   f) a power model: ________________________

5. <u>What do you conclude judging by the results obtained</u>? Do you know now the best model?

Appendix C

Activity on Statistical Charts

Graphing Data Activity

We can use different types of graphs to represent data. The objective is that the graph organizes the data in a way that makes it easier/faster to understand. We will look at frequency charts, bar charts, dot plots, pie charts, and histograms.

The following sets of data are test scores. Just glancing at this data it is not immediately obvious if one class did better than the other. Once this data has been organized using charts and statistics, it will be easier to compare the sets.

Class 1: 64, 75, 77, 60, 92, 77, 82, 40, 77, 80, 37, 50, 92, 90, 90, 80, 72, 92, 97, 92

Class 2: 50, 40, 87, 60, 100, 97, 72, 60, 54, 87, 80, 92, 54, 80, 72, 75, 92, 82, 44, 87

Scores of 90 and above are A’s, 80’s are B’s, 70’s are C’s, 60’s are D’s, and below 60 are F’s.

First, start organizing this data by listing it in ascending order:

Class 1:

Class 2:

Just looking at low scores and high scores, which class appears to have done better?

A frequency chart for grades reports the number of times each grade occurs. The frequency chart for Class 1 grades is filled in below. **Fill in the chart for Class 1 grades.**

Class 1 Grades	Frequency
a	7
b	3
c	5
d	2
f	3
Total	20

Class 2 Grades	Frequency
a	
b	
c	
d	
f	
Total	

Which class appears to have done better based on the information in the frequency charts? Explain.

Look at the frequency charts

- It only needs 2 columns: grades and frequency
- A total is usually given

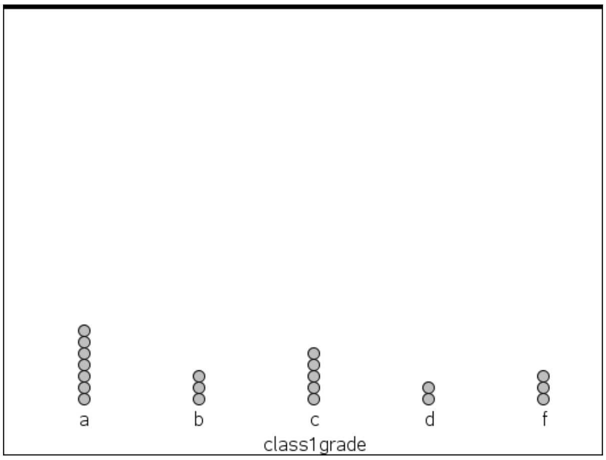
A dot plot will simply give a visual representation of the data in the frequency chart. We will construct dot plots of the data using our TI-Nspire software.

First put in four columns of data as shown here.

	A class1score	B class1grade	C class2score	D class2grade	E
◆					
1	37	f	40.	f	
2	40	f	44.	f	
3	50	f	50.	f	
4	60	d	54.	f	
5	64.	d	54	f	
6	72.	c	60	d	
7	75.	c	60	d	
8	77	c	72	c	
9	77	c	72	c	
10	77	c	75.	c	
11	80	b	80.	b	
12	80	b	80	b	

C1 40.00

Next, insert a new screen in data and statistics. It will probably look like something like the graph on the left. If you click on the bottom and add the variable class1grade, it should produce the dot plot shown on the left.



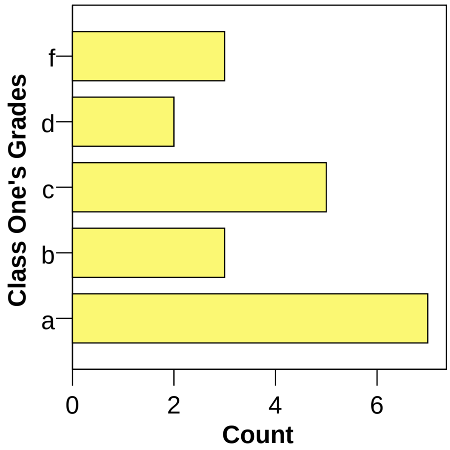
If you right click on the screen it will give you a choice of a bar chart or a pie chart. We will be graphing those soon. **Cut and paste the dot plot for class 2 grades below.**

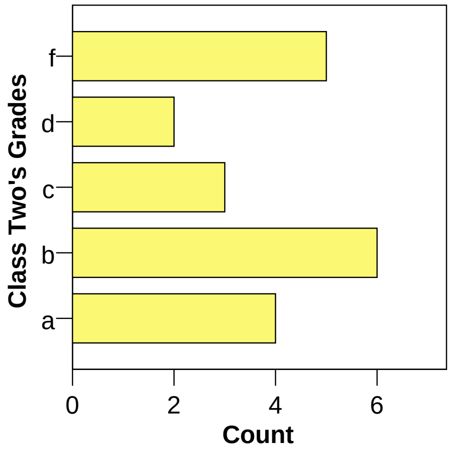
Note that the dot plots need dots that are the same size and equally spaced.

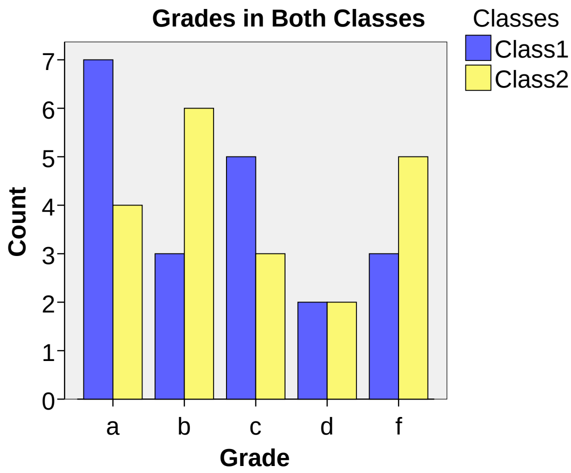
Now we will look at the bar charts. They need:

- Equally spaced bars that have the same width (some books do not always space them – oops!)
- Chart name and axis labels

The dot plots do not typically show the count because you can count the dots. The first two bar charts below show grades for just one test. The third one is a side-by-side chart that makes it easier to compare them.





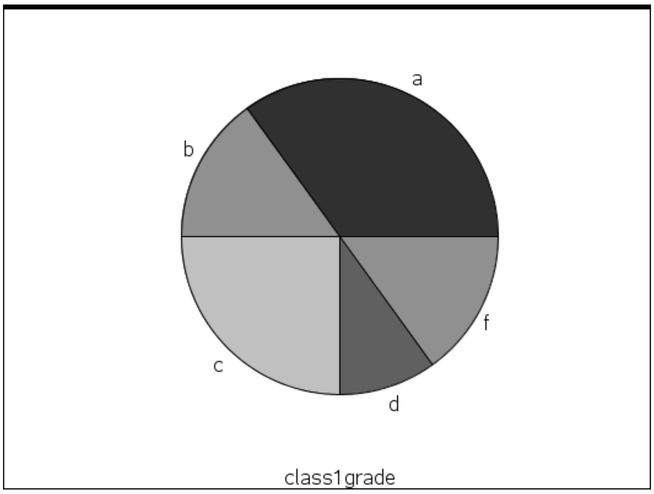


The first two have horizontal bars and the third one has vertical bars. You will cut and paste vertical bar chart for class1 and 2 grades below. To construct them all that you need to do is right click on the screens with the dot plots and choose bar chart.

Now we will look at the Pie Charts. They need:

- Chart name, labels and percents,
- Slices that correspond to the percent of the circle.

To construct them, start by right clicking on the screens with the dot plots or bar charts and choose pie chart. The one for the grades for class 1 is shown below. Notice, it is not showing any percents. To get the percents, first calculate the percent value for each grade. Next, go back to screen, click on the action key (arrow), click on Insert Text, type in the desired percent, and drag it to the corresponding slice. That is how the graph on the right was constructed.



Cut and paste the pie chart for class 2 grades below (with percents).

Note: When constructing pie charts by hand, the size of the slices needs to be calculated. To prepare the students for this calculation, you could start by asking:

How many degrees are in a circle?

If a category is 50% how many degrees will it get? 25%? 10%? 37%

For 50% it is 180 degrees, of ½ of 360 degrees, or 0.50(360). The idea is to get the students to realize that they need to multiply the percent by 360 degrees to get the number of degrees in the central angle of the slice. Once they do this warn them to be careful to put the percent in the chart after they have measured the angles. Often times students will put in the degrees instead of the percents.

The charts constructed so for used the categorical data of the grades. **How are the frequency, dot, bar, and pie charts similar and different?**

Now we will move on to charts that use the continuous data of the test scores.

We will start with histograms and frequency polygons

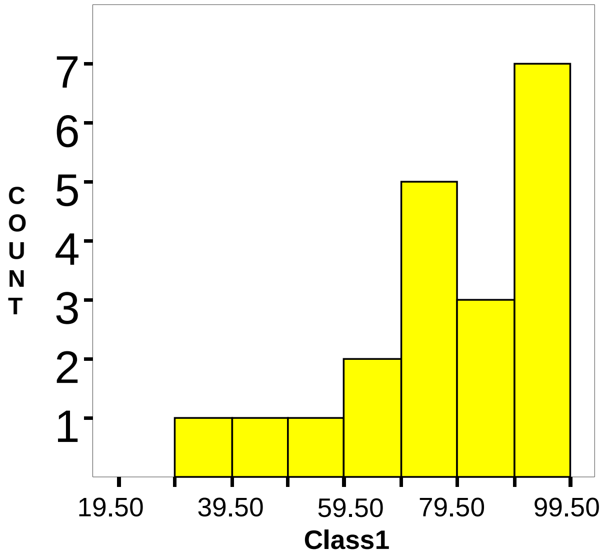
Look at the histogram below for test1 scores.

- For histograms, you have equal-sized categories (width of the bars).

  What is the size of each of the categories in the histogram?

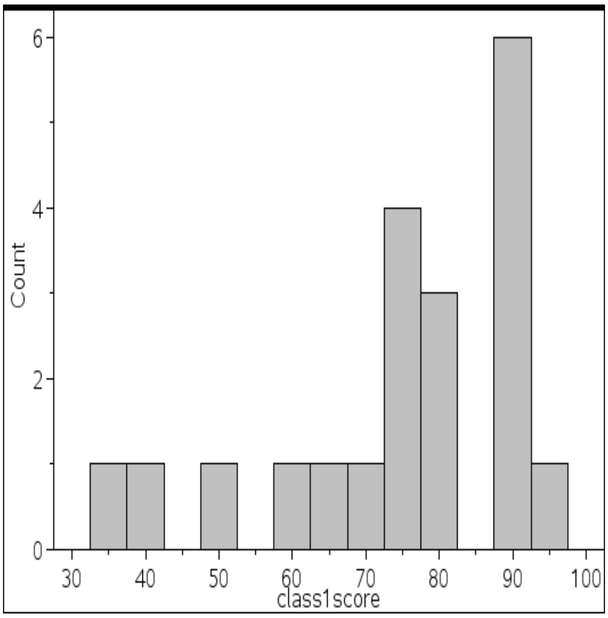
  What would be the size of the five categories in the bar charts if we artificially forced them into a histogram? (Hint 1: The B category would have endpoints of 79.5 to 89.5. Hint 2: 100 should be included in the category for A.)

- In histograms, the categories border each other, so the bars will touch (as long as the categories are not empty).

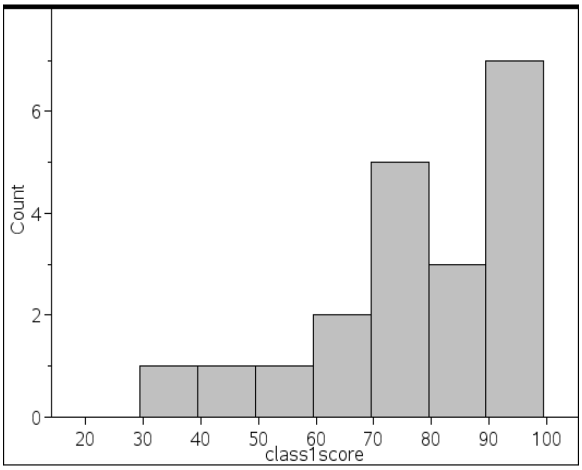


Now we will use the TI-Nspire software to create a histogram for Class 1 Scores.

Insert a new screen in data and statistics. Click on the bottom and add the variable class1score. Left click on the screen and choose Histogram, it should produce the histogram shown below on the left. This looks very different from the histogram above, so we are going to work to make it look more like that one.



Click on the Plot Properties button above the histogram (second from left), choose histogram properties, Bin Settings. Change the width to 10 and put in one of your interval endpoints for alignment (such as 99.5). You may have to pull down on your count axis to get the top of your largest bar in the picture and you may need to pull over on your and score axis to show the 20. Now you should have something like:



The shape is the same as the previous graph. I do not know how to change where the tick marks are placed. Please let me know if you figure it out.

Create a similar histogram for the class 2 data and put it below.

Below are histograms for two different tests. Which class do you think did better? Explain giving at least 2 reasons.